

CORONAL HEATING, WEAK MHD TURBULENCE, AND SCALING LAWS

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ABSTRACT

Long-time high-resolution simulations of the dynamics of a coronal loop in Cartesian geometry are carried out, within the framework of reduced magnetohydrodynamics (RMHD), to understand coronal heating driven by the motion of field lines anchored in the photosphere. We unambiguously identify MHD anisotropic turbulence as the physical mechanism responsible for the transport of energy from the large scales, where energy is injected by photospheric motions, to the small scales, where it is dissipated. As the loop parameters vary, different regimes of turbulence develop: strong turbulence is found for weak axial magnetic fields and long loops, leading to Kolmogorov-like spectra in the perpendicular direction, while weaker and weaker regimes (steeper spectral slopes of total energy) are found for strong axial magnetic fields and short loops. As a consequence we predict that the scaling of the heating rate with axial magnetic field intensity B_0 , which depends on the spectral index of total energy for given loop parameters, must vary from $B_0^{3/2}$ for weak fields to B_0^2 for strong fields at a given aspect ratio. The predicted heating rate is within the lower range of observed active region and quiet-Sun coronal energy losses.

Subject headings: MHD — Sun: corona — Sun: magnetic fields — turbulence

1. INTRODUCTION

In this Letter we solve, within the framework of RMHD in Cartesian geometry, the Parker field-line tangling (coronal heating) problem (Parker 1972, 1988). We do this via long simulations at high resolutions, introducing hyperresistivity models to attain extremely large Reynolds numbers. We show how small scales form and how the coronal heating rate depends on the loop and photospheric driving parameters, and we derive simple formulae that may be used in the coronal heating context for other stars.

Over the years a number of numerical experiments have been carried out to investigate coronal heating, with particular emphasis on exploring how photospheric field line tangling leads to current sheet formation.

Mikić et al. (1989) and Hendrix & Van Hoven (1996) first carried out simulations of a loop driven by photospheric motions using a Cartesian approximation (a straightened out loop bounded at each end by the photosphere) imposing a time-dependent alternate direction flow pattern at the boundaries. A complex coronal magnetic field results from the photospheric field line random walk, and although the field does not, strictly speaking, evolve through a sequence of static force-free equilibrium states (the original Parker hypothesis), magnetic energy nonetheless tends to dominate kinetic energy in the system. In this limit the field is structured by current sheets elongated along the axial direction, separating quasi-two-dimensional (2D) flux tubes that constantly move around and interact. Galsgaard & Nordlund (1996) carried out similar studies, while Longcope & Sudan (1994) focused on the current sheet formation process within the RMHD approximation, also used in the simulations by Dmitruk & Gómez (1999). The results from these studies agreed qualitatively among themselves, in that all simulations display the development of field-aligned current sheets. However, estimates of the dissipated power and its scal-

ing characteristics differed largely, depending on the way in which extrapolations from low to large values of the plasma conductivity of the properties such as inertial range power-law indices were carried out. 2D numerical simulations of incompressible MHD with magnetic forcing (Einaudi et al. 1996; Georgoulis et al. 1998; Dmitruk et al. 1998; Einaudi & Velli 1999) showed that turbulent current sheets dissipation is distributed intermittently, and that the statistics of dissipation events, in terms of total energy, peak energy, and event duration displays power laws not unlike the distribution of observed emission events in optical, ultraviolet, and X-ray wavelengths of the quiet solar corona.

More recently, full 3D sections of the solar corona with a realistic geometry have been simulated by Gudiksen & Nordlund (2005). While this approach has advantages when investigating the coronal loop dynamics within its neighboring coronal region, numerically modeling a larger part of the solar corona drastically reduces the number of points occupied by the coronal loops. Thus, these simulations have not been able to shed further light on the *physical mechanism* responsible for the coronal heating.

In § 2 we introduce the coronal loop model and the simulations we have carried out; in § 3 we describe our numerical results, and in § 4 we give simple scaling arguments to understand the magnetic energy spectral slopes. This will lead to a quantitative asymptotic estimate of the coronal loop heating rate, and of its scaling with the axial magnetic field, photospheric velocity amplitude and coronal loop length.

2. THE MODEL

A coronal loop is a closed magnetic structure threaded by a strong axial field, with the footpoints rooted in the photosphere. This makes it a strongly anisotropic system, as measured by the relative magnitude of the Alfvén velocity $v_A \sim 1000 \text{ km s}^{-1}$ compared to the typical photospheric velocity $u_{ph} \sim 1 \text{ km s}^{-1}$. This means that the relative amplitude of the Alfvén waves that are launched into the corona is very small. The loop dynamics may be studied in a simplified geometry, neglecting any curvature effect, as a “straightened out” Cartesian box, with an orthogonal square cross section of size ℓ_\perp , and an axial length L embedded in an axial homogeneous uniform magnetic

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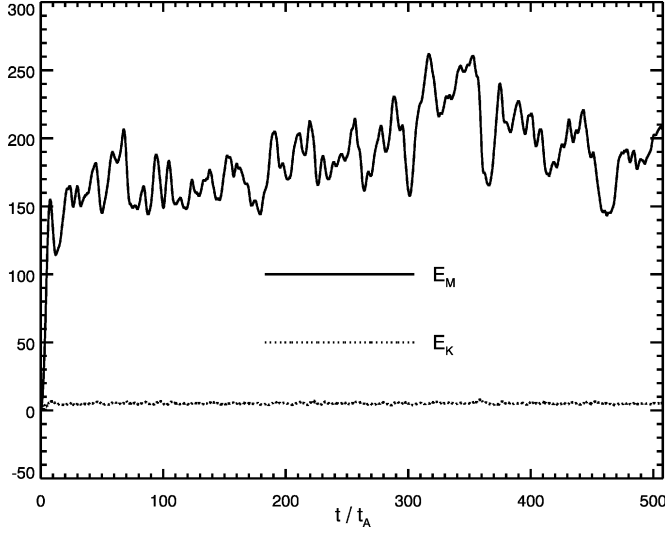


FIG. 1.—High-resolution simulation with $v_A/u_{ph} = 200$, $512 \times 512 \times 200$ grid points and $Re_1 = 800$. Magnetic (E_M) and kinetic (E_K) energies as a function of time ($\tau_A = L/v_A$ is the axial Alfvénic crossing time).

field $\mathbf{B}_0 = B_0 \mathbf{e}_z$. This system may be described by the RMHD equations (Kadomtsev & Pogutse 1974; Strauss 1976; Montgomery 1982): introducing the velocity and magnetic field potentials φ and ψ , $\mathbf{u}_\perp = \nabla \times (\varphi \mathbf{e}_z)$, $\mathbf{b}_\perp = \nabla \times (\psi \mathbf{e}_z)$, and vorticity and current, $\omega = -\nabla_\perp^2 \varphi$, $j = -\nabla_\perp^2 \psi$ the nondimensioned RMHD system is given by

$$\frac{\partial \psi}{\partial t} = v_A \frac{\partial \varphi}{\partial z} + [\varphi, \psi] + \frac{(-1)^{n+1}}{Re_n} \nabla_\perp^{2n} \psi, \quad (1)$$

$$\frac{\partial \omega}{\partial t} = v_A \frac{\partial j}{\partial z} + [j, \psi] - [\omega, \varphi] + \frac{(-1)^{n+1}}{Re_n} \nabla_\perp^{2n} \omega. \quad (2)$$

As characteristic quantities we use the perpendicular length of the computational box ℓ_\perp , the typical photospheric velocity u_{ph} , and the related crossing time $t_\perp = \ell_\perp/u_{ph}$. The equations have been rendered dimensionless using velocity units for the magnetic field (the density in the loops ρ is taken to be constant) and normalizing by u_{ph} . Then the nondimensioned Alfvén speed v_A in equations (1)–(2) is given by the ratio v_A/u_{ph} between the dimensional velocities. The Poisson bracket of two functions g and h is defined as $[g, h] = \partial_x g \partial_y h - \partial_y g \partial_x h$, where x, y are transverse coordinates across the loop while z is the axial coordinate along the loop. A simplified diffusion model is assumed and Re_n is the Reynolds number, with n the hyperdiffusion index (*dissipativity*): for $n = 1$ ordinary diffusion is recovered.

The computational box spans $0 \leq x, y \leq 1$, and $0 \leq z \leq L$, with $L = 10$ corresponding to an aspect ratio equal to 10. As boundary conditions at the photospheric surfaces ($z = 0, L$) we impose a velocity pattern intended to mimic photospheric motions, made up of two independent large spatial scale projected convection cell flow patterns. The wavenumber values k excited are all those in the range $3 \leq k \leq 4$, and the average injection wavenumber is $k_{in} \sim 3.4$.

3. RESULTS

Plots of the rms magnetic and kinetic energies as a function of time, together with the dissipation due to currents, vorticity, as well as the integrated Poynting flux, are shown in Figures 1 and 2. As a result of the photospheric forcing, energy in the

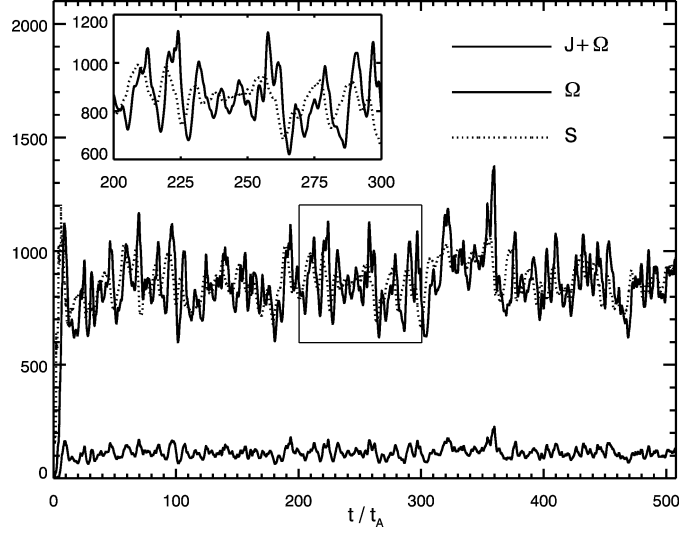


FIG. 2.—Same simulation of Fig. 1. The integrated Poynting flux S dynamically balances the ohmic (J) and viscous (Ω) dissipation. Inset shows a magnification of total dissipation and S for $200 \leq t/t_A \leq 300$.

magnetic field first grows with time, until it dominates over the kinetic energy by a large factor, before oscillating, chaotically, around a stationary state. Fluctuating magnetic energy E_M is ~ 35 times bigger than kinetic energy E_K .

The same generic features are seen in the rms current and vorticity dissipation, although the time dependence of the signal is more strongly oscillating. The ohmic dissipation rate J is ~ 6.5 times viscous dissipation Ω . The Poynting flux, on average, follows the current dissipation (there is no accumulation of energy in the box); however, a detailed examination shows that the dissipation time series tends to lag the Poynting flux, with notable decorrelations around significant dissipation peaks. The spatial configuration of the currents which corresponds to a snapshot at a given time is displayed in Figure 3 (Plate 1). The currents collapse into warped, torn sheets that extend almost completely along the loop. The current peaks are embedded within the 2D sheetlike structures, corresponding to an anisotropic structure for the turbulence, in agreement with previous results.

A dimensional analysis of equations (1)–(2) shows that the only free nondimensional quantity is $f = \ell_\perp v_A / Lu_{ph}$. We fix $L/\ell_\perp = 10$ and vary the ratio of the Alfvén speed to photospheric convection speed v_A/u_{ph} . Both runs with standard second-order dissipation ($n = 1$), as well as hyperdiffusion ($n = 4$), have been carried out to obtain extended inertial ranges in the resulting spectra.

The power spectrum of total energy in the simulation box, once a statistically stationary state has been achieved, depends strongly on the ratio v_A/u_{ph} . This was first found in simulations by Dmitruk et al. (2003) devoted to understanding how anisotropic regimes of MHD turbulence depend on boundary driving strength, with whom our numerical work is in broad agreement.

The total energy spectrum, for values of $v_A/u_{ph} = 50, 200, 400$, and 1000 is shown in Figure 4, together with fits to the inertial range power law. As v_A/u_{ph} increases, the spectrum steepens visibly (note that the hump at the high wave-vector values for the runs with large v_A/u_{ph} is a feature, the bottleneck effect, which is well known and documented in spectral simulations of turbulence with the hyperdiffusion used here; see, e.g., Falkovich 1994), with the slopes ranging from -2 to almost -3 . At the same time while total energy increases, the ratio of the mean magnetic field over the axial Alfvén velocity

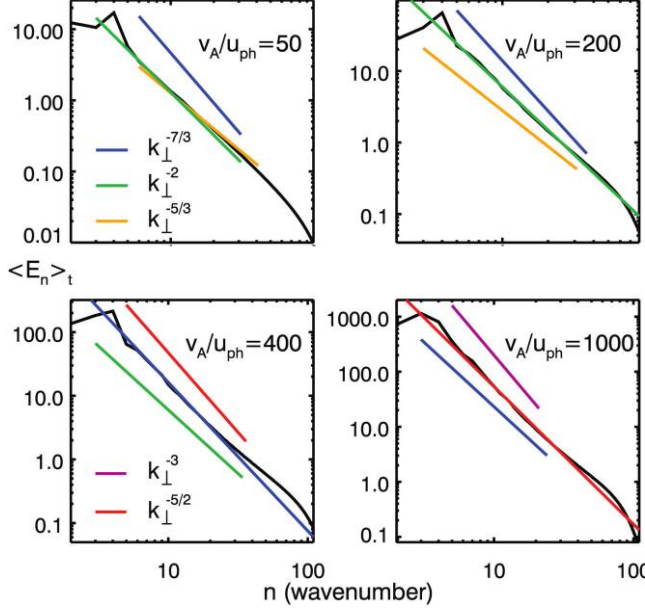


FIG. 4.—Time-averaged total energy spectra for simulations with $v_A/u_{ph} = 50, 200, 400$, and 1000 . Hyperdiffusion ($n = 4$) has been used with $Re_A = 3 \times 10^{20}, 10^{20}, 10^{19}$, and 10^{19} , respectively, and a grid with $512 \times 512 \times 200$ points.

decreases, in good accordance with the theory. This steepening, which may be interpreted as both the effect of inertial line-tying of the coronal magnetic field and the progressive weakening of nonlinear interactions as the magnetic field is increased, has a strong and direct bearing on the coronal heating scaling laws.

4. DISCUSSION

A characteristic of anisotropic MHD turbulence is that the cascade takes place mainly in the plane orthogonal to the DC magnetic guide field (Shebalin et al. 1983). Consider then the anisotropic version of the Iroshnikov-Kraichnan (IK) theory (Sridhar & Goldreich 1994; Goldreich & Sridhar 1997). Dimensionally the energy cascade rate may be written as $\rho \delta z_\lambda^2 / T_\lambda$, where δz_λ is the rms value of the Elsässer fields $\mathbf{z}^\pm = \mathbf{u}_\perp \pm \mathbf{b}_\perp$ at the perpendicular scale λ , where because the system is magnetically dominated $\delta z_\lambda^+ \sim \delta z_\lambda^-$. ρ is the average density and T_λ is the energy transfer time at the scale λ , which is greater than the eddy turnover time $\tau_\lambda \sim \lambda / \delta z_\lambda$ because of the Alfvén effect (Iroshnikov 1964; Kraichnan 1965).

In the classical IK case, $T_\lambda \sim \tau_\lambda (\tau_\lambda / \tau_A)^2$. This corresponds to the fact that wave packets interact over an Alfvén crossing time (with $\tau_\lambda > \tau_A$), and the collisions follow a standard random walk in energy exchange. In terms of the number of collisions N_λ that a wave packet must suffer for the perturbation to build up to order unity, for IK $N_\lambda \sim (\tau_\lambda / \tau_A)^2$.

More generally, however, as the Alfvén speed is increased the interaction time becomes smaller, so that turbulence becomes weaker and the number of collisions required for efficient energy transfer scales as

$$N_\lambda = \left(\frac{\tau_\lambda}{\tau_A} \right)^\alpha \quad \text{with } \alpha > 2, \quad (3)$$

where α is the scaling index (note that $\alpha = 1$ corresponds to standard hydrodynamic turbulence), so that

$$T_\lambda \sim N_\lambda \tau_A \sim \left(\frac{v_A}{L} \right)^{\alpha-1} \left(\frac{\lambda}{\delta z_\lambda} \right)^\alpha. \quad (4)$$

Integrating over the whole volume, the energy transfer rate becomes

$$\epsilon \sim \ell_\perp^2 L \rho \frac{\delta z_\lambda^2}{T_\lambda} \sim \ell_\perp^2 L \rho \left(\frac{L}{v_A} \right)^{\alpha-1} \frac{\delta z_\lambda^{\alpha+2}}{\lambda^\alpha}. \quad (5)$$

Considering the injection scale $\lambda \sim \ell_\perp$, equation (5) becomes

$$\epsilon \sim \ell_\perp^2 L \rho \frac{\delta z_{\ell_\perp}^2}{T_{\ell_\perp}} \sim \frac{\rho \ell_\perp^2 L^\alpha}{\ell_\perp^\alpha v_A^{\alpha-1}} \delta z_{\ell_\perp}^{\alpha+2}. \quad (6)$$

On the other hand, the energy injection rate is given by the Poynting flux integrated across the photospheric boundaries: $\epsilon_{in} = \rho v_A \int d\mathbf{u}_{ph} \cdot \mathbf{b}_\perp$. Considering that this integral is dominated by energy at the large scales, due to the characteristics of the forcing function, we can approximate it with

$$\epsilon_{in} \sim \rho \ell_\perp^2 v_A u_{ph} \delta z_{\ell_\perp}, \quad (7)$$

where the large scale component of the magnetic field can be replaced with δz_{ℓ_\perp} because the system is magnetically dominated.

The last two equations show that the system is self-organized because both ϵ and ϵ_{in} depend on δz_{ℓ_\perp} , the rms values of the fields \mathbf{z}^\pm at the scale ℓ_\perp : the internal dynamics depends on the injection of energy and the injection of energy itself depends on the internal dynamics via the boundary forcing. Another aspect of self-organization results from our simulations: the perpendicular magnetic field develops few spatial structures along the axial direction z , and in the nonlinear stage its topology substantially departs from the mapping of the boundary velocity pattern that characterizes its evolution during the linear stage. These and other features will be discussed more in depth in A. F. Rappazzo et al. (2007, in preparation).

In a stationary cascade the injection rate (7) is equal to the transport rate (6). Equating the two yields for the amplitude at the scale ℓ_\perp ,

$$\frac{\delta z_{\ell_\perp}^*}{u_{ph}} \sim \left(\frac{\ell_\perp v_A}{Lu_{ph}} \right)^{\alpha/(\alpha+1)}. \quad (8)$$

Substituting this value in equations (6) or (7), we obtain for the energy flux

$$\epsilon^* \sim \ell_\perp^2 \rho v_A u_{ph}^2 \left(\frac{\ell_\perp v_A}{Lu_{ph}} \right)^{\alpha/(\alpha+1)}, \quad (9)$$

where $v_A = B_0 / (4\pi\rho)^{1/2}$. This is also the dissipation rate and hence the *coronal heating scaling*. The crucial parameter here is $f = \ell_\perp v_A / Lu_{ph}$ because the scaling index α (eq. [3]), on which the strength of the stationary turbulent regime depends, must be a function of f itself. The relative amplitude of the turbulence $\delta z_{\ell_\perp}^* / v_A$ is a function of f , and as f increases the effect of *line-tying* becomes stronger, decreasing the strength of turbulent interactions (wave-packet collision efficiency becomes subdiffusive) so that α increases above 2. The ratio $\delta z_{\ell_\perp}^* / v_A$ can also

be interpreted as the rms value of the Parker angle Θ_p and is given by

$$\langle \Theta_p \rangle \sim \frac{\delta z_{\perp}^*}{v_A} \sim \left(\frac{\ell_{\perp}}{L} \right)^{\alpha/(\alpha+1)} \left(\frac{u_{ph}}{v_A} \right)^{1/(\alpha+1)}. \quad (10)$$

This is actually an estimate of the average inclination of the magnetic field lines, while the rms value of the shear angle between neighboring field lines is at least twice that given by equation (10), not considering that close to a current sheet an enhancement of the orthogonal magnetic field is observed (which leads to a higher value for the angle).

Numerical simulations determine the remaining unknown nondimensional dependence of the scaling index α on f . The power-law slopes of the total energy spectra shown in Figure 5 are used to determine α . Identifying, as usual, the eddy energy with the band-integrated Fourier spectrum $\delta z_{\lambda}^2 \sim k_{\perp} E_{k_{\perp}}$, where $k_{\perp} \sim \ell_{\perp}/\lambda$, from equation (5) we obtain

$$E_{k_{\perp}} \propto k_{\perp}^{-(3\alpha+2)/(\alpha+2)}, \quad (11)$$

where for $\alpha = 1$ the $-5/3$ slope for the “anisotropic Kolmogorov” spectrum is recovered, and for $\alpha = 2$ the -2 slope for the anisotropic IK case. At higher values of α correspond steeper spectral slopes up to the asymptotic value of -3 .

In Figure 5 we plot the values of α determined in this way, together with the resulting power dependence $\alpha/(\alpha+1)$ of the amplitude (8) and of the energy flux (9) on the parameter f . The other power dependences are easily obtained from this last one, e.g., for the energy flux (9) the power of the axial Alfvén speed v_A is given by $1 + \alpha/(\alpha+1)$, so that in terms of the magnetic field B_0 it scales as $B_0^{3/2}$ for weak fields and/or long loops, to B_0^2 for strong fields and short loops.

Dividing equation (9) by the surface ℓ_{\perp}^2 we obtain the energy flux per unit area $F = \epsilon^*/\ell_{\perp}^2$. Taking for example a coronal loop 40,000 km long, with a number density of 10^{10} cm^{-3} , $v_A =$

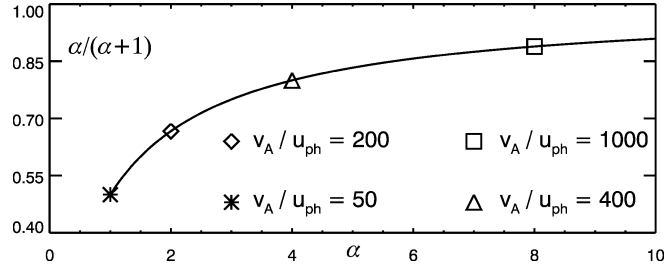


FIG. 5.—Solid line: Exponent $\alpha/(\alpha+1)$ as a function of α . Symbols: Values of α corresponding to different values of v_A/u_{ph} , at fixed $L/\ell_{\perp} = 10$.

2000 km s⁻¹, and $u_{ph} = 1 \text{ km s}^{-1}$, [for these parameters we can estimate a value of $\alpha/(\alpha+1) \sim 0.95$], which models an active region loop, we obtain $F \sim 5 \times 10^6 \text{ ergs cm}^{-2} \text{ s}^{-1}$ and a Parker angle (eq. [10]) $\langle \Theta_p \rangle \sim 4^\circ$. On the other hand, for a coronal loop typical of a quiet-Sun region, with a length of 100,000 km, a number density of 10^{10} cm^{-3} , $v_A = 500 \text{ km s}^{-1}$, and $u_{ph} = 1 \text{ km s}^{-1}$ [for these parameters we can estimate a value of $\alpha/(\alpha+1) \sim 0.7$], we obtain $F \sim 7 \times 10^4 \text{ ergs cm}^{-2} \text{ s}^{-1}$ and $\langle \Theta_p \rangle \sim 0.9^\circ$.

In summary, with this Letter we have shown how coronal heating rates in the Parker scenario scale with coronal loop and photospheric driving parameters, demonstrating that field line tangling can supply the coronal heating energy requirement. We also predict that there is no universal scaling with axial magnetic field intensity, a feature that can be tested by observing weak field regions on the Sun, or the atmospheres of other stars with differing levels of magnetic activity.

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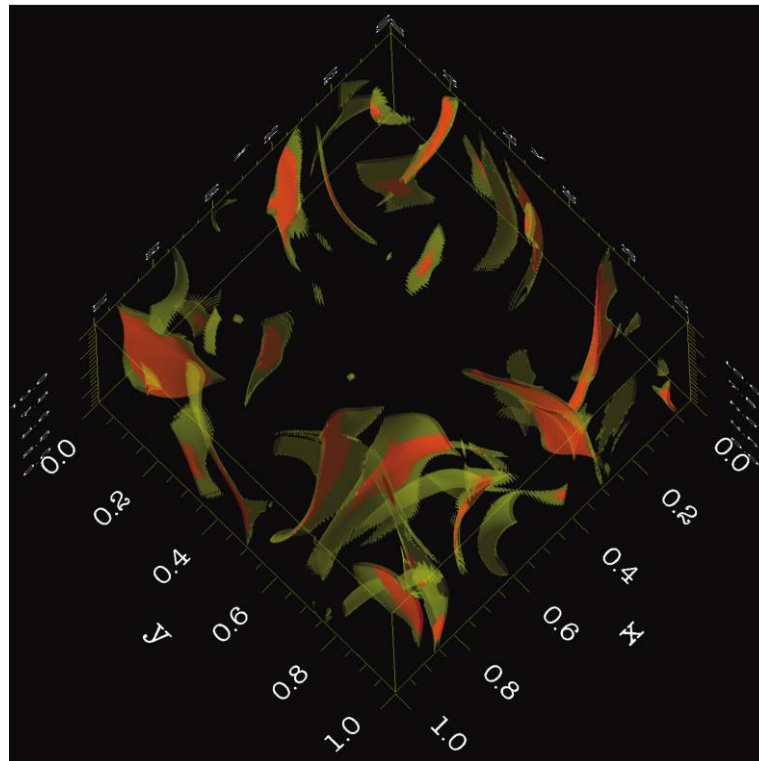
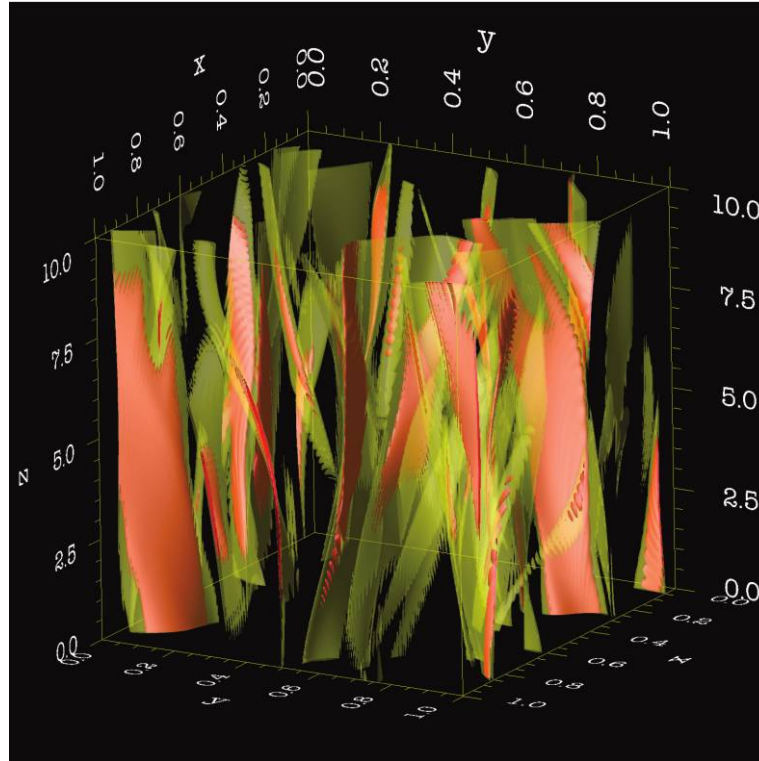


PLATE 1

FIG. 3.—*Top*: Side view of two isosurfaces of the squared current at a selected time for a numerical simulation with $v_A/u_{ph} = 200$, $512 \times 512 \times 200$ grid points and a Reynolds number $Re_1 = 800$. The isosurface at the value $j^2 = 2.8 \times 10^5$ is represented in partially transparent yellow, while red displays the isosurface with $j^2 = 8 \times 10^5$, well below the value of the maximum of the squared current that at this time is $j^2 = 8.4 \times 10^6$. Note that the red isosurface is always nested inside the yellow one and appears pink in the figure. The computational box has been rescaled for an improved viewing, but the aspect ratio of the box is 10; i.e., the axial length of the box is 10 times bigger than its orthogonal length. *Bottom*: Top view of the same two isosurfaces using the same color display. The isosurfaces are extended along the axial direction, and the corresponding filling factor is small.